EXACT ANALYSIS OF SHIELDED MICROSTRIP LINES AND BILATERAL FINLINES

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ABSTRACT

An exact analysis is presented for shielded microstrip and bilateral finlines. The method is based on a function theoretic approach to solve a set of functional equations in the Fourier transform domain, representing the independent excitation of LSM and LSE modes in the system without strip or fin conductors. The solution is obtained in the form of highly convergent system of algebraic equations, which allow accurate calculation of fields and electrical parameters of these lines at any given frequency.

I. INTRODUCTION

Microstrip lines, being an important element of microwave integrated circuits, were extensively investigated both theoretically and experimentally/1-8/. However, it seems that the problem has not yet been solved completely as remarked in/9/. Most methods of analysis suggested in the literature suffer from serious limitations and usually include assumptions that may lead to considerable uncercainty in the obtained results. Thus, most design calculations are till now performed using quasi-static results of Wheeler and others. More rigorous hybrid mode analysis are done either numerically or using mathematical methods leading to sets of equations the convergence of which is not high enough to insure sufficiently accurate results at a reasonable effort or to allow a physical understanding of the problem. Alternatives to microstrip lines for use in integrated circuits at higher microwave and millimeter wave frequencies were introduced in the form of unilateral and bilateral finline configurations/10-12/. These lines were investigated by several authors using different techniques /13-16/. Due to the fact that finlines are always mounted in rectangular waveguides, there has been a general tendency to treat them as modified forms of ridged guides. This tendency reflected on the mathematical methods used, which are in fact very similiar to those used for ridged guides and waveguide discontinuities. The waveguiding properties of the gap between the fins, regardless of the waveguide housing itself were almost overlooked. In fact finlines can support guided waves even when the housing is removed altogether, as fields are concentrated in the gap regions. In the following, a method based on modified Wiener-Hopf technique is applied to shielded microstrip and bilateral finlines without side-walls. The formulation of the problem is exact and no assumptions were made during the solution. The high rate of convergence obtained allowed essentially accurate determination of the electrical parameters .

II. FORMULATION OF THE PROBLEM

Consider the dual structures shown on fig. 1. , comprising symmetrical strip and bilateral finlines with two symmetrically located shields. The width of the strip or the gap width of the fins are denoted by w, the thickness of the dielectric layer is 2d, and d_0 is the distance between the dielectric surface and the shield. The relative permittivity and permeability of the dielectric are ε_r, μ_r respectively, while those of free space are ε_0, μ_0.

Fundumental mode fields of the microstrip and bilateral finlines correspond to electric wall symmetry in the microstrip and magnetic wall symmetry in the finline configurations of fig.1. w.r.t. the plane at the middle of the dielectric layer thickness, leading to the basic models of fig.2. The cartesian axes x, y, z are chosen as shown.

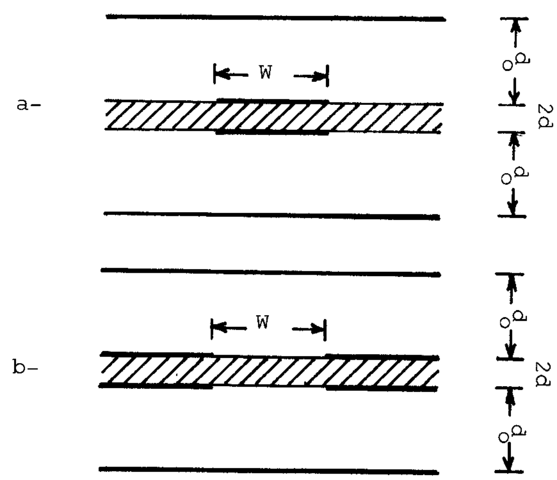

Fig.1. Symmetrical strip(a) and bilateral finline(b) configurations.

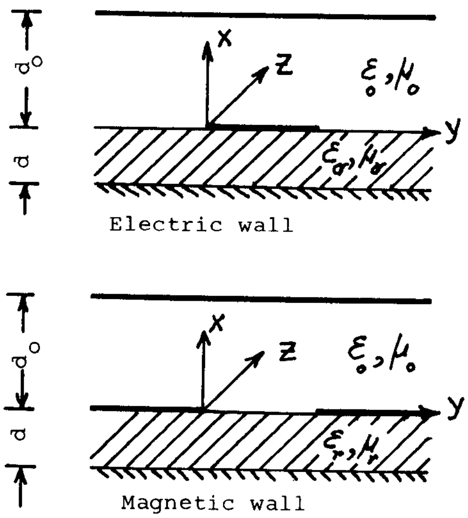

Fig.2.Microstrip line and bilateral finline configurations considered in the paper.

Considering the structure without the strip or fin conductors,which is in fact a dielectric loaded parallel plate waveguide, we assume a surface current distribution $J_y(y,z),J_z(y,z)$ to flow over the dielectric surface.The surface currents will excite a field with electric field components tangential to the dielectric surface given by $E_y(0,y,z),E_z(0,y,z)$.

Considering fields with dependence on time t and the longitudinal coordinate z of the form $\exp(i\gamma z-i\omega t)$, where γ is a real propagation constant and ω is the angular frequency, field and current components are expressed in terms of thier Fourier transforms ,defined for a function f(y) as

$$\hat{f}(\alpha)=\int_{-\infty}^{+\infty} f(y)e^{-i\alpha y}dy \qquad (1)$$

Next we introduce a set of functions $U_1(\alpha)$, $U_2(\alpha)$, $F_1(\alpha)$, $F_2(\alpha)$ as linear combinations of the Fourier transforms of surface currents and tangential electric field components:

$$\begin{aligned} U_1&=-\alpha\hat{J}_y(\alpha)+\gamma\hat{J}_z(\alpha)\\ U_2&=\gamma\hat{J}_y(\alpha)+\alpha\hat{J}_z(\alpha)\\ F_1&=-\alpha\hat{E}_y(\alpha)+\gamma\hat{E}_z(\alpha)\\ F_2&=\gamma\hat{E}_y(\alpha)+\alpha\hat{E}_z(\alpha)\end{aligned} \qquad (2)$$

Functions F_1,F_2 can be shown to be related to U_1,U_2 through the following set of equations:

$$\begin{aligned} -i\omega\varepsilon_o X_1(\alpha)F_1(\alpha)&=U_1(\alpha)\\ i\,X_2(\alpha)F_2(\alpha)&=\omega\mu_o U_2(\alpha)\end{aligned} \qquad (3)$$

X_1,X_2 are the transforms of inverse Green's functions for sources of LSM and LSE wave types respectively in the loaded guide without strips or fins.They have thier zeroes, rather than poles, coinciding with the propagation constants of these modes.

Explicit expressions for X_1,X_2 have the form

$$X_1=\frac{\operatorname{Coth}R_o d_o}{R_o}+\varepsilon_r\frac{\operatorname{Coth}Rd}{R}$$

$$X_2=R_o\operatorname{Coth}R_o d_o+\frac{1}{\mu_r}R\operatorname{Coth}Rd$$

for electric wall symmetry(microstrip case), and

$$X_1=\frac{\operatorname{Coth}R_o d_o}{R_o}+\varepsilon_r\frac{\operatorname{Tanh}Rd}{R}$$

$$X_2=R_o\operatorname{Coth}R_o d_o+\frac{1}{\mu_r}R\operatorname{Tanh}Rd\ ,$$

for magnetic wall symmetry(finline case), where

$$R_o^2=\alpha^2+\gamma^2-k_o^2\ ,\quad R^2=\alpha^2+\gamma^2-k^2\ ,$$
$$k_o^2=\omega^2\varepsilon_o\mu_o\ ,\quad k^2=\varepsilon_r\mu_r k_o^2\ .$$

III.FIELDS IN THE MICROSTRIP LINE

The set of functional relations (3) can be used for the solution of the problem of propagation in the microstrip line.

If the strip is assumed to be thin and ideally conducting,then the following boundary conditions should be satisfied:

$$\left.\begin{aligned}E_y(0,y)&=0\\E_z(0,y)&=0\end{aligned}\right\}\ \text{for}\quad 0<y<W\ ,$$

$$\left.\begin{aligned}J_y(y)&=0\\J_z(y)&=0\end{aligned}\right\}\ \text{for}\quad y<0\ ,\ y>W\ . \qquad (4)$$

Boundary conditions (4) will reflect on the properties of the functions F_1,F_2,U_1,U_2 as follows:

a- U_1,U_2 will be entire functions having algebraic behavior on the upper half of the α-plane .

b- F_1,F_2 can be expressed as

$$F_1=F_1^-(\alpha)\pm e^{i\alpha W}F_1^-(-\alpha)\ ,$$

$$F_2=F_2^-(\alpha)\mp e^{i\alpha W}F_2^-(-\alpha)\ .$$

Functions F_1^-,F_2^- are regular in the lower half-plane and have algebraic behavior for large α . Upper signs refer to modes with symmetrical longitudinal current distribution on the strip, while lower signs to anti-symmetrical modes .

Therefore for the microstrip line the following functional equations can be written:

$$\frac{i}{\omega\varepsilon_o X_1} U_1(\alpha)=F_1^-(\alpha)\pm e^{i\alpha W}F_1^-(-\alpha)$$
$$\frac{\omega\mu_o}{i X_2} U_2(\alpha)=F_2^-(\alpha)\mp e^{i\alpha W}F_2^-(-\alpha) \qquad (5)$$

Equations (5) allow solution using modified Wiener-Hopf technique/17/.Omitting details, the solution for symmetrical modes can be written in the form

$$F_1^-(\alpha)=\frac{P}{X_1^-}\left\{1-\sum_{n=0}^{\infty}\frac{\xi_n}{\alpha-\alpha_n}A_n\right\}$$
$$F_2^-(\alpha)=\frac{Q}{X_2^-}\left\{1-\sum_{n=1}\frac{\zeta_n}{\alpha-\alpha_n}B_n\right\} \qquad (6)$$

P,Q are yet undetermined constants and α_n are the poles of X_1, X_2 ,which coincide except for the zero-order pole of X_1,given by

$$\alpha_o=\sqrt{k^2-\gamma^2} \quad .$$

These poles represent the propagation constants of waveguide modes in the regions above and under the strip. X_1^-, X_2^- are functions, regular and having no roots on the lower half-plane, resulting from the factorisation of X_1, X_2 . Coefficients A_n, B_n satisfy the following systems of equations:

$$A_n=1+\sum_{m=0}\frac{\xi_m}{\alpha_n+\alpha_m}A_m \quad , \ n=0,1,2,..$$
$$B_n=1+\sum_{m=1}\frac{\zeta_m}{\alpha_n+\alpha_m}B_m \quad , \ n=1,2,.. \qquad (7)$$

Coefficients ξ_n, ζ_n are given by

$$\xi_n=\frac{\mathrm{Res}\, X_1^-(\alpha_n)}{X_1^-(-\alpha_n)}e^{i\alpha_n W}, \quad -\zeta_n=\frac{\mathrm{Res}X_2^-(\alpha_n)}{X_2^-(-\alpha_n)}e^{i\alpha_n W}$$

For propagating microstrip modes all α_n , with the exception of α_o ,are imaginary and are given by

$$\alpha_n \left\langle \begin{array}{l}\sqrt{k^2-\gamma^2-(n\pi/d)^2} \ , \\ \sqrt{k_o^2-\gamma^2-(n\pi/d_o)^2} \ .\end{array}\right.$$

When the strip width is not too small, systems (7) are exponentially convergent and can be effectively solved by iteration techniques to practically any required degree of accuracy .

It should be noted,that till this point fields of LSM and LSE types were treated quite independently.This is actually the main advantage of the introduction of the variables $F_{1,2}$, $U_{1,2}$.

However,fields in microstrip lines should be of the hybrid type and LSM,LSE fields are necessarily coupled .This coupling is actually present as P,Q have to satisfy certain conditions,necessary to achieve physically proper field behavior .

Once F_1,F_2,U_1,U_2 are determined,the physical variables J_y, J_z, E_y, E_z can be obtained through transformations inverse to (2).These inverse linear transformations are singular at the points

$$\alpha=\pm i\gamma \quad .$$

Since $\hat{J}_y,\hat{J}_z$ are entire functions,the constants P and Q must be chosen such that

$$U_1(\pm i\gamma)\pm i\,U_2(\pm i\gamma)=0,$$

which is equivalent to

$$F_1^-(\pm i\gamma)\pm i\,F_2^-(\pm i\gamma)=0. \qquad (8)$$

Conditions (8) lead to a set of two homogeneous linear equations in P and Q.For non-vanishing fields the determinant must be equal to zero. This condition determines the possible values of the propagation constant γ and the coupling of fields,as expressed by the ratio P/Q.

IV. WAVE PROPAGATION IN FINLINES

The presence of fins will impose boundary conditions,dual to (4). Thus,surface currents J_y, J_z vanish on the gaps,while tangential electric field components E_y, E_z must vanish on the fins:

$$\left.\begin{array}{l}J_y(y)=0\\ J_z(y)=0\end{array}\right\} \quad \text{for} \quad 0<y<W,$$
$$\left.\begin{array}{l}E_y(0,y)=0\\ E_z(0,y)=0\end{array}\right\} \quad \text{for} \quad y<0,\ y>W. \qquad (9)$$

To satisfy boundary conditions (9),the following properties must be prescribed to the F and U functions:

a- F_1,F_2 are entire functions having algebraic behavior in the upper half-plane.

b- U_1,U_2 for finline modes with anti-symmetrical longitudinal currents should be represented in the form

$$U_1=U_1^-(\alpha)+e^{i\alpha W}U_1^-(-\alpha),$$
$$U_2=U_2^-(\alpha)-e^{i\alpha W}U_2^-(-\alpha),$$

where U_1^-, U_2^- are functions,regular in the lower half-plane.

Taking into consideration the properties a,b ,functional equations (3) can be written

$$-i\omega\varepsilon_o X_1(\alpha)F_1(\alpha)=U_1^-(\alpha)+e^{i\alpha W}U_1^-(-\alpha),$$

$$\frac{iX_2(\alpha)}{\omega\mu_0} F_2(\alpha)=U_2^-(\alpha) - e^{i\alpha W} U_2^-(-\alpha), \qquad (10)$$

Equations (10) represent the complete mathematical problem of the bilateral finline. They can be solved in a way similiar to that used for (5). Taking into account the edge conditions, this solution can be written

$$U_1^-(\alpha)= P\, X_1^-(\alpha) \left\{ 1 + \sum_{n=1} \frac{\xi_n}{\alpha-\gamma_n} A_n \right\}$$
$$U_2^-(\alpha)= Q\, X_2^-(\alpha) \left\{ 1 - \sum_{n=1} \frac{\zeta_n}{\alpha-\sigma_n} B_n \cdot \right\} \qquad (11)$$

P,Q are yet undetermined constants. X_1^-,X_2^- are the "minus" functions resulting from the factorisation of X_1,X_2. Coefficients ξ_n, ζ_n are given by

$$\xi_n = \frac{[X_1^+(\gamma_n)]^2}{X_1'(\gamma_n)} e^{i\gamma_n W}, \quad \zeta_n = \frac{[X_2^+(\sigma_n)]^2}{X_2'(\sigma_n)} e^{i\sigma_n W}$$

γ_n, σ_n are the roots of X_1,X_2 on the upper half-plane, while the coefficients A_n,B_n are determined from the following systems of equations

$$A_n = 1 - \sum_{m=1} \frac{\xi_m}{\gamma_n+\gamma_m} A_m, \quad n=1,2,\ldots,$$
$$B_n = 1 + \sum_{m=1} \frac{\zeta_m}{\sigma_n+\sigma_m} B_m, \quad n=1,2,\ldots \qquad (12)$$

Except for very narrow-gap finlines, systems (12) are highly convergent due to the presence of exponential factors in ξ_n, ζ_n, γ_n, σ_n being imaginary for most surface waves. Coefficients A_n,B_n can practically be calculated to any required degree of accuracy at a reasonable computational effort.

Coupling of LSE and LSM fields is established as in the case of microstrip line through relations between P,Q, which have to be satisfied in order to eliminate singularities in the transforms of the tangential electric field components and surface currents. These relations are written as

$$U_1^-(\pm i\gamma) \pm i\, U_2^-(\pm i\gamma) = 0.$$

Since U_1^-,U_2^- are proportional to P,Q, these constants satisfy two homogeneous linear equations. The simultaniety of these equations requires a vanishing determinant. Thus the possible values of γ are determined as the roots of this determinant and the ratio P/Q, which is the measure of LSE-LSM coupling, may be calculated.

V. NUMERICAL RESULTS AND PHYSICAL INTERPRETATION.

For checking of the effectiveness of the suggested method computations have been performed for a microstrip line with parameters

$$\varepsilon_r=8.875,\quad \mu_r=1.0,\quad d_0/d=10,$$
$$W/d=0.5,1,2,4.$$

and for bilateral finline with parameters:

$$\varepsilon_r=9.7,\quad \mu_r=1.0,\quad d_0/d=5.0$$
$$W/d=0.5,1,2,4.$$

Factorisation of the meromorphic functions X_1,X_2 was achieved by the standard procedure of expansion in infinite products involving poles and roots. The infinite sets of equations (7),(12), being nearly diagonal, were solved by iterative procedure showing very quick convergence. The results of computation of the effective dielectric constant

$$\varepsilon_{eff.}=(\gamma/k_0)^2,$$

and the wave impedance are shown on fig.3,4. The definition used for the wave impedance of the microstrip line is similiar to that adopted at low frequencies i.e. the ratio of the voltage at the center of the strip to the total longitudinal current. For finlines the wave impedance is taken as the ratio of the integral over the y-component of the electric field over the gap to the total longitudinal current on one of the fins.

In the case of microstrip line approximate calculations using only one term n=0 in the sum over A_n are also shown as dashed lines. Coincidence of the approximate and exact values over a considerable part of the frequency interval indicates high rate of convergence of the systems of equations, especially at high frequencies. Moreover it shows that the field propagates along the microstrip line essentially in a multiple reflection mode as in rectangular and dielectric guides. The field under the strip is essentially the sum of two plane TEM waves propagating at angles $\pm\varphi$, where φ is given by

$$\varphi=\cos^{-1}(\gamma/k),$$

to the z-axis and are reflected from the strip edges. In the case of narrow strips or at low frequencies this picture is distorted as the evanescent modes at the edges will couple together significantly.

In bilateral finlines the propagation of non-attenuating waves is possible only when all poles of X_1,X_2, which coincide with the propagation constants of waveguide modes in the regions $y<0$, $y>W$, are imaginary. Otherwise, the excitation of these modes will necessarily lead to loss of power through radiation sidewards and to the decay of the main wave in the longitudinal direction. These poles form two sets:

$$\alpha_n \begin{cases} \sqrt{k_0^2-\gamma^2-(n\pi/d_0)^2}, & n=0,1,2,\ldots \\ \sqrt{k^2-\gamma^2-[(n-\tfrac{1}{2})\pi/d]^2}, & n=1,2,\ldots \end{cases}$$

Therefore the allowed values of γ are limited to the range

$$k^2>\gamma^2> \max.\left\{k_0^2, k^2-(\pi/2d)^2\right\}.$$

For this range of γ all propagation constants of LSM and LSE modes in the gap region are imaginary, except for the lowest order LSE mode corresponding to the first root of X_2, This root denoted by $\tilde{\sigma}_1$,can be either real or imaginary.Depending on the value of $\tilde{\sigma}_1$, two modes of propagation in finlines can be distinguished.When $\tilde{\sigma}_1$ is imaginary the field in the gap region has a quasi-static character and all coefficients ξ_n, ζ_n are real.When $\tilde{\sigma}_1$ is real,the field propagates in a waveguide mode,guided by multiple reflections of the surface wave from the fin edges,where conditions of total reflection exist as all waveguide modes are evanescent.computations have shown ,that the two modes are possible. Quasi-static mode dominates at low frequencies while the waveguide mode is dominant at high frequencies.

Calculations revealed a curious behavior of the dispersion curves of finlines at different gap widths in the transition region between quasi-static and waveguide modes. It was found that these curves,regardless of the gap width,intersect at a common point on the line representing the dispersion characteristics of the surface wave mode corresponding to $\tilde{\sigma}_1$ .This can be explained by the fact that the effect of the width of the gap on the dispersion characteristics is different in the two regions.Thus,in the quasi-static mode smaller gaps tend to lower the phase velocity due to field concentration in the dielectric.In the waveguide mode this effect is reversed as wider gaps tend to decrease the phase velocity towards the value for the free surface wave velocity.This effect is analogous to the effect of width in rectangular guides.Therefore,the family of dispersion curves at different widths should have an intersection point where the effect is reversed .

Following this analysis it must be remarked,that the bilateral finline is senstive to geometrical imperfections violating the symmetry of the field,e.g. relative displacement of the gaps.In this case the fundumental TEM mode in the dielectric filled waveguides between the fins will be excited, leading to loss of power in the side directions in finlines without walls or to strong coupling to the walls if they are present.

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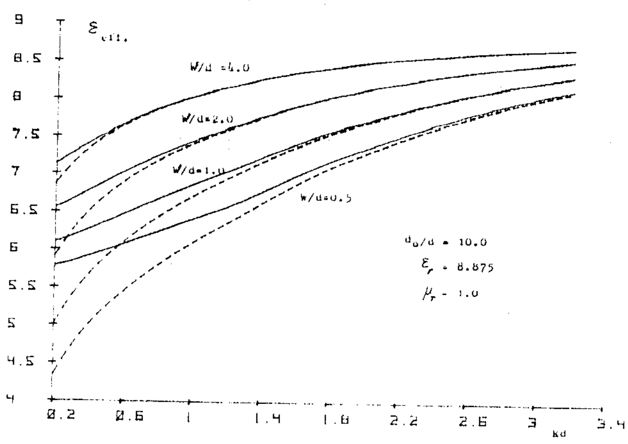

Fig.3-a.Dispersion characteristics of microstrip line.

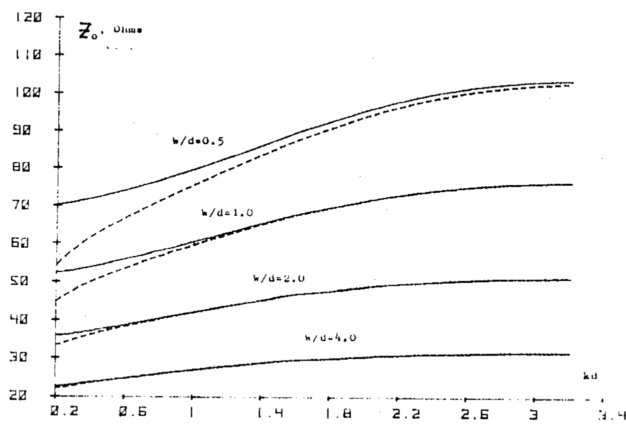

Fig.3-b.Dependence of microstrip line wave impedance upon the normalised frequency kd .

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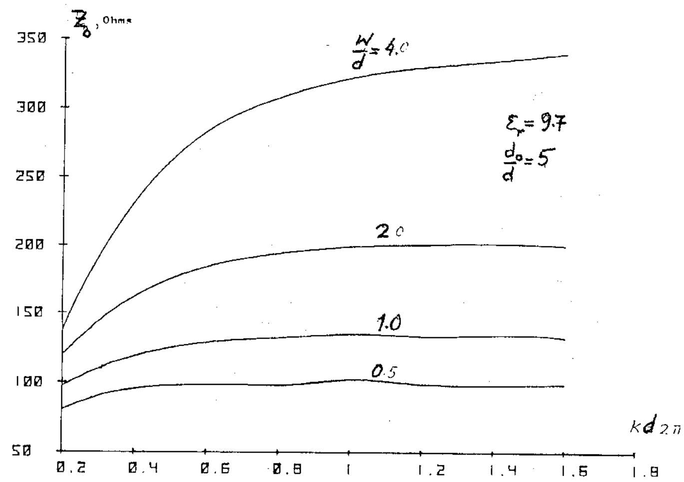

Fig.4-b.Variation of bilateral finline wave impedance with the normalised frequency parameter kd.

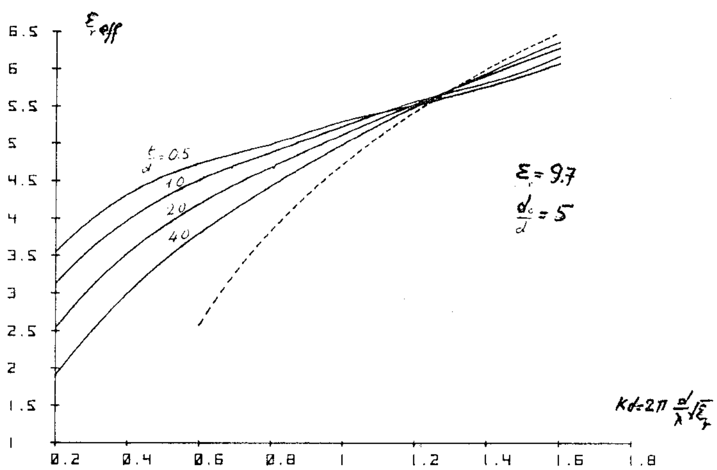

Fig.4-a.Dispersion characteristics of bilateral finline showing intersection on the free surface wave line (dashed line)